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Preliminary studies on a variant of TSP for servicing printers and copiers

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Preliminary Studies on a Variant of TSP for Servicing Printers and Copiers *

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Abstract. We analyze a case study and we model it as a variant of the Traveling Salesman Problem. It is characterized by multiple time windows, variable service duration, priority constraints, and appointments with customers. The definition of the feasible region differs from the classical one: a maximum duration is imposed to the tour, and some customers may be neglected. The number of customers to visit is fixed as the maximum number given all constraints: it is found by solving a further optimization problem. The objective function to minimize is the sum of the time required to serve this number of customers. We propose a mathematical formulation for this problem and we solve it on two real instances. We compare the solutions obtained with the ones implemented by the firm.

Keywords: Case Study, Traveling Salesman Problem.

JEL Classification Numbers: M37, M31, C61.

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1 Introduction

The analysis at the basis of this paper is motivated by the need of solving a case study, that can be modeled as a variant of the Traveling Salesman Problem (TSP). The TSP consists in finding the minimum length circuit to be traveled by a vehicle, visiting a set of customers exactly once. The objective of the problem is usually to minimize the total length of the tour. In the literature several analysis are devoted to the TSP. Among the variants studied, the ones considered most often include time windows constraints ([1, 2, 3, 4]).

In this paper, we tackle a problem characterized by multiple time windows, variable service duration, priority constraints, and appointments fixed with customers. The definition of the feasible region differs from the typical one: a maximum duration is imposed to the tour, and it may include only a subset of customers. The number of customers to visit is fixed as the maximum number given all constraints: it is found by solving a further optimization problem. The overall objective function to minimize is the total cost borne for serving this number of customers.

In this paper we propose a mathematical formulation for this problem and we solve it on two real instances. We compare the solutions obtained with the ones implemented by the firm, both in a static and in a dynamic framework.

In Section 2 we describe the case study considered and in Section 3 we formulate the model. We present the computational experiments in Section 4. Finally, we report some conclusions and hints for future research are in Section 5.

2 The case study

Pellegrini SpA is a firm that operates in the field of Information & Communication Technology. Nowadays it is one of the most important Italian firm in this field: 120 employees, 15 millions of euros of yearly turnover, 8000 customers in the whole Italy. Figure 1 shows the geographical distribution of customers, and their density in the different areas of Italy and in the Veneto Region.

Pellegrini SpA's core business is papery document management. The products it commercializes are printers, copiers, faxes, scanners. The firm believes that one of the main strengths of its provision relationship with customers is the constant operativity of the equipment. Hence Pellegrini SpA offers to its customers not only high quality products and qualified commercial consulting, but also a good technical service.

In this paper we study the method according to which the activity of service men is organized. Currently, customers are partitioned in clusters, each of which is assigned to a service man. Every day each service man receives from the central office a list of customers of its cluster with a service request. The service man decides autonomously the order to follow for performing these services. His objective is completing the highest number of services in his working day.

Each costumer may indicate its temporal availability: the service must necessarily be performed in this interval. Furthermore, the central office and the customer may agree on the time and the date of the service by fixing an appointment.

[Italy]

[Veneto Region]

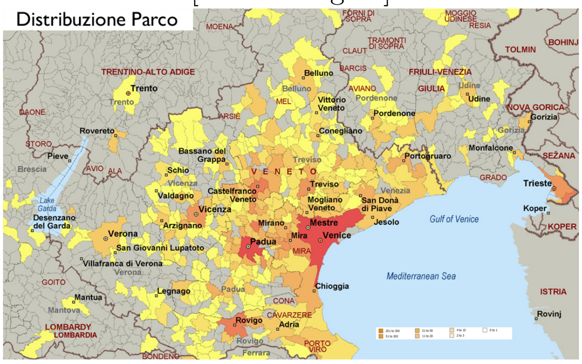

Figure 1: Geographical distribution of Pellegrini SpA's customers, and their density in different areas.

Service requests may have different urgency: important customers must be visited as shortly as possible after they request the service. Moreover, services on completely unusable pieces of equipment are more urgent than the others.

The service man is free to take the lunch break whenever he prefers. If in that moment he is close enough to his basis (usually his home address), i.e., less than 20 minutes driving, he is supposed to have lunch there. Otherwise he can choose any bar or restaurant in the neighborhood and he can obtain a refunding of 10 Euros.

3 The model

We model the problem described in Section 2 as a variant of the TSP. The notation and the decision variables needed for formalizing it as an integer linear program are described hereafter.

For each service man and for each working day let:

- $I_1 = \{0, 1, ..., I, 2I+1\}$ be the set of service requests that may performed. 0 and $2I+1$ are the basis of service man: the tour has to start from 0 in the morning and to return to $2I+1$ at the end of the working day.
- $I_2 = \{I+1, ..., 2I\}$ be the set of fictitious service requests that represent the lunch break. $I+i$ is the lunch break if it takes place after the service request i.
- $I^A \subset I_1$ be the set of service requests for which the central office fixed an appointment.
- d_i be the estimated duration of service request i, for all $i \in I_1 \cup I_2$; $d_i = D$, for all $i \in I_2$, with D duration of a lunch break; $d_0 = d_{2I+1} = 0$.
- t_{ij} be the time required to travel from service request i to service request j, for all $i, j \in I_1 \cup I_2$;
  $t_{ij} = \infty$ if a) $i, j \in I_2$, or b) $i \in I_1$ and $j \in I_2, j \neq i+I$, or c) $j \in I_1$ and $i \in I_2, i = j+I$;
  $$t_{i,i+I} = \begin{cases} 0, & \text{if } t_{i0} \geq T, \\ t_{i0}, & \text{otherwise,} \end{cases} \qquad t_{i+I,j} = \begin{cases} t_{ij}, & \text{if } t_{i0} \geq T, \\ t_{0j}, & \text{otherwise,} \end{cases}$$
  for all $i \in I_1$, where T is the maximum traveling time for which the service man is supposed to go at his basis for lunch: if the time distance is short enough, the service man must go at his basis for lunch. Correspondingly, if he got lunch at his basis, he starts from there his trip to the next service.
- c_{ij} be the cost of traveling from service request i to service request j, for all $i, j \in I_1 \cup I_2$;
  $c_{ij} = \infty$ if a) $i, j \in I_2$, or b) $i \in I_1$ and $j \in I_2, j \neq i+I$, or c) $j \in I_1$ and $i \in I_2, i = j+I$;
  $$c_{i,i+I} = \begin{cases} C, & \text{if } t_{i0} \geq T, \\ c_{i0}, & \text{otherwise,} \end{cases} \qquad c_{i+I,j} = \begin{cases} c_{ij}, & \text{if } t_{i0} \geq T, \\ c_{0j}, & \text{otherwise,} \end{cases}$$
  for all $i \in I_1$, where C is the amount refunded for a lunch in a bar or restaurant.

- $[e_i^h, l_i^h]$ be the time window h of the service request i, $i \in I_1 \cup I_2$, $h \in \{1,2\}$;
  $e_i^1 = 12$ am, $l_i^1 = 2$ pm if $i \in I_2$; $e_i^2 = 11$ pm, $l_i^2 = 12$ pm if $i \in I_2$ or $i \in I_1$ a unique time window is available.

- $p_i \in \{0,1\}$ be the priority of the service request i, $i \in I_1$;
  $p_i = 1$, if the service request i is urgent, $p_i = 0$, otherwise.

- $[ae_i, al_i]$ be the time interval in which the service request i can start if an appointment is fixed, for all $i \in I^A$. If $i \in I^A$ then the service request i is urgent, that is if an appointment is fixed then $p_i = 1$ (appointment $\Rightarrow$ priority).

- D_{max} be the maximum duration of a working day.

- lb be the duration of a lunch break.

- P_i be the penalty for a time unit delay with respect to the appointment fixed for the service request i, $i \in I_A$.

- M large constant.

- τ earliest possible starting time of the working day.

The decision variables are the following:

- $$x_{ij}^h = \begin{cases} 1\,, & \text{if the service request } j \text{ is performed after the service request } i \\ & \quad \text{during the time window } h, \\ 0\,, & \text{otherwise,} \end{cases}$$

  for all $i, j \in I_1 \cup I_2$, $h \in \{1,2\}$.

- s_i starting time of the service request i, for all $i \in I_1 \cup I_2$.

- r_i delay at the service request i with respect to the appointment, for all $i \in I^A$.

The objective of the problem is to determine a minimum cost path, provided that the maximum number of service requests during the working day is performed, while respecting all constraints.

The model $(minC)$ can be formulated as follows

$$\min \sum_{i,j \in I_1 \cup I_2} \{c_{ij} \sum_{h \in \{1,2\}} x_{ij}^h\} + \sum_{i \in I^A} r_i P_i, \tag{1}$$

subject to

$$s_0 \geq \tau; \tag{2}$$

$$s_j - (s_i + d_i + t_{ij}) \leq M(1 - \sum_{h \in \{1,2\}} x^h_{ij}),\ \forall\ i, j \in I_1 \cup I_2; \tag{3}$$

$$s_j - (s_i + d_i + t_{ij}) \geq M(\sum_{h \in \{1,2\}} x^h_{ij} - 1), \forall\ i, j \in I_1 \cup I_2; \tag{4}$$

$$\sum_{j \in I_1 \cup I_2} x^h_{ji} e^h_i \leq s_i \leq \sum_{j \in I_1 \cup I_2} x^h_{ji} l^h_i,\ \forall\ i \in I_1 \cup I_2, \quad h \in \{1,2\}; \tag{5}$$

$$(\sum_{j \in I_1 \cup I_2, h \in \{1,2\}} x^h_{ij} - 1) p_i = 0,\ \forall\ i \in I_1 \cup I_2; \tag{6}$$

$$\sum_{j \in I_1 \cup I_2, h \in \{1,2\}} x^h_{ji} \leq 1,\ \forall\ i \in I_1 \cup I_2 \setminus \{0\}; \tag{7}$$

$$\sum_{j \in I_1 \cup I_2, h \in \{1,2\}} x^h_{ij} \leq 1,\ \forall\ i \in I_1 \cup I_2 \setminus \{2I+1\}; \tag{8}$$

$$\sum_{j \in I_1 \cup I_2} \sum_{h \in \{1,2\}} x^h_{ji} = \sum_{j \in I_1 \cup I_2} \sum_{h \in \{1,2\}} x^h_{ij},\ \forall\ i \in I_1 \cup I_2 \setminus \{0, 2I+1\}; \tag{9}$$

$$\sum_{j \in I_1 \cup I_2, h \in \{1,2\}} x^k_{0j} = 1; \tag{10}$$

$$\sum_{j \in I_1 \cup I_2, h \in \{1,2\}} x^k_{j0} = 0; \tag{11}$$

$$\sum_{j \in I_1 \cup I_2, h \in \{1,2\}} x^k_{j2I+1} = 1; \tag{12}$$

$$\sum_{j \in I_1 \cup I_2, h \in \{1,2\}} x^k_{2I+1j} = 0; \tag{13}$$

$$\sum_{i \in I_1, j \in I_2, h \in \{1,2\}} x^h_{ij} = 1; \tag{14}$$

$$s_{2I+1} - s_0 \leq D_{max} + lb; \tag{15}$$

$$\sum_{i,j \in I_1 \cup I_2, h \in \{1,2\}} x^h_{ij} = \overline{N}; \tag{16}$$

$$ae_i \leq s_i,\ \forall\ i \in I^A; \tag{17}$$

$$s_i - al_i \leq r_i,\ \forall\ i \in I^A; \tag{18}$$

$$s_i \geq 0, \; \forall \; i \in I_1 \cup I_2; \tag{19}$$

$$x_{ij}^h \in \{0,1\} \; \forall \; i,j \in I_1 \cup I_2, \; h \in \{1,2\}; \tag{20}$$

$$r_i \geq 0 \; \forall \; i \in I^A. \tag{21}$$

The objective function (1) includes the total traveling costs and the penalties for delay. Constraints (2), (3) and (4) describe the relations among the duration of service requests and the traveling times. Constraints (5) and (6) ensure that solutions fulfill time windows and priorities. Constraints (7) and (8) ensure that each service request is performed at most once. Moreover if the service man arrives at the customer for performing service request i, then he has to leave the customer afterwards (9). Again, the service man has to leave his starting basis 0 (10) without getting back to it (11), and he has to return to $2I + 1$ (12) without leaving afterwords (13). The service man has to take exactly one lunch break in a working day (14). The constraint (15) makes the tour to respect the maximum duration of the working day. Constraint (16) imposes that the number of service requests performed in the working day is $\overline{N}$, i.e., the maximum number of service requests that it is actually possible to perform. Constraints (17) ensures that the service does not start before the time of the appointment, if any appointment is fixed, and constraints (18) defines the delay cumulated. Finally constraints (19), (20) and (21) state that decision variables are binary (whenever suitable) and nonnegative.

The maximum number of service requests that can be performed in a working day, $\overline{N}$, is the optimal value function of a following problem that can be modeled as follows ($maxN$):

$$\max \sum_{i,j \in I_1 \cup I_2, h \in \{1,2\}} x_{ij}^h, \tag{22}$$

subject to (2)–(15), (19)–(20)
and

$$ae_i \leq s_i \leq al_i, \; \forall \; i \in I^A. \tag{23}$$

Constraints (23) impose the tour to respect all appointments.

4 Experimental analysis

For testing our model we considered the service requests performed by two service men in the third week of September 2010.

In agreement with the firm we set the following values for parameters:

- maximum duration of a working day: 8 hours and 30 minutes.
- duration of lunch break: 30 minutes.

- penalty for a time unit delay with respect to an appointment: 28.4 Euros (as half an hour driving).
- earliest possible starting time of the working day: 8 am.
- $c_{ij} = t_{ij} \cdot 40$ Euros per hour $+$ Km for going from i to $j \cdot 0.28$ Euros per Km

We solved *maxN* and *minC* using XPRESS optimizer version 20.00.11 on a Intel Core 2 Duo CPU at 1.86 GHz and with 2.00Gb of Ram.

For speeding up the solution of *maxN* we added the constraint

$$\sum_{i,j \in I_1 \cup I_2, h \in \{1,2\}} x_{ij}^h \leq \hat{N} \tag{24}$$

and we progressively increased $\hat{N}$ until a feasible solution could not be found within 60 seconds.

Once fixed $\overline{N}$ in this way we solved *minC* on the feasible region found.

4.1 Instance 1

We analyzed the service requests performed by the first service man in the week considered (instance 1).

In Table 1 we describe the solution implemented by Pellegrini SpA. The services performed are 24, seven of which are completed in the same day in which the requests are made. The total cost is 351.92 Euros, borne in four working days, with 126 minutes of overtime. Due to overtime this solution is not feasible for our model.

Table 1: Solution implemented by Pellegrini SpA - instance 1

day	available	performed	cost
1	13	6	117.47
2	13	6	94.38
3	10	6	69.93
4	6	6	70.14
5	0	0	0.00

In Table 2 we report the solution that we obtained simulating the situation in which we fix the service order in the morning, using the estimated duration of the service requests. The services performed are 24. Fixing the order in the morning it is not possible to visit any customer in the day in which he makes the request. The total cost is 383.27 Euros, borne in five days. A posteriori, considering real service duration, this solution ends up being infeasible due to 38 minutes of overtime.

The real service duration is often significantly different from the estimated one. For proposing an efficient feasible solution we tackle the problem dynamically: the service order is optimized after each service request is performed. In this way it is also possible to serve customers as soon as they make a request. In Table 3 we report the solution obtained:

Table 2: Solution proposed if the service order is fixed in the morning - instance 1

day	available	performed	cost
1	13	7	79.31
2	12	6	96.13
3	9	6	97.99
4	5	3	55.90
5	2	2	53.94

Table 3: Solution proposed if the order is optimized after each service - instance 1

day	available	performed	cost
1	13	6	70.96
2	13	6	92.31
3	10	6	110.43
4	6	5	99.09
5	1	1	35.7

all the 24 services are performed, and six of them are performed in the day in which the requests are made. The total cost is 408.49 Euros. This cost is higher than the cost of the solution that was actually implemented, but the opposite would hold if we considered the cost of overtime. We do not quantify this cost following the indication of the firm that aims at avoiding it completely.

4.2 Instance 2

We analyze the service requests performed by the second service man in the week considered (instance 2). Following the structure of Section 4.1, we report the solution implemented by Pellegrini SpA, the one obtained fixing the service order in the morning, and the one dynamically re-optimized in Table 4, 5, and 6, respectively. The last one implies the lowest total cost (724.76 Euros), while the costs entailed by the first two are 850.04 and 932.2 Euros respectively. In all the cases the services performed are 28 in five working days. Only the solution actually implemented implies overtime (38 minutes); it allows to serve 11 customers in the day in which they make the request. The corresponding number in the dynamically re-optimized solution is eight.

Table 4: Solution implemented by Pellegrini SpA - instance 2

day	available	performed	cost
1	13	6	167.30
2	13	7	148.30
3	12	5	178.92
4	10	6	139.73
5	6	6	215.79

Table 5: Solution proposed if the service order is fixed in the morning - instance 2

day	available	performed	cost
1	13	7	144.8
2	12	6	127.24
3	12	5	202.2
4	10	6	182.74
5	6	4	275.22

Table 6: Solution proposed if the order is optimized after each service - instance 2

day	available	performed	cost
1	13	7	147.97
2	12	8	133.18
3	10	5	145.64
4	8	5	224.32
5	5	3	73.55

5 Conclusions

In this paper we have presented an integer linear program for solving a case study proposed by an Italian firm. We have modeled it a variant of the TSP in which the feasible region is identified through a further formulation. We have tested our model on two real instances. We have compared the optimal solution obtained both in a static and a dynamic framework with the one actually implemented.

The solution actually implemented is often infeasible, implying some overtime. While the solution obtained in the static framework may result in being a posteriori infeasible, the one obtained in the dynamic framework allows to increase the efficiency of the system guaranteeing feasibility.

Future research will be devoted to an extensive experimental analysis, and possibly to the development of alternative solution approaches.

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